

Problem 8

In the following problems, find the limit of the given sequence as $n \rightarrow \infty$.

$$\frac{(n!)^2}{(2n)!}$$

Solution

Take the limit as $n \rightarrow \infty$.

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{(n!)^2}{(2n)!} &= \lim_{n \rightarrow \infty} \frac{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 \cdot n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}{2n(2n-1)(2n-2)(2n-3)(2n-4) \cdots (n+1)n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1} \\ &= \lim_{n \rightarrow \infty} \frac{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}{2n(2n-1)(2n-2)(2n-3)(2n-4) \cdots (n+1)} \\ &= \lim_{n \rightarrow \infty} \frac{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}{2^n n \left(n - \frac{1}{2}\right) \left(n - 1\right) \left(n - \frac{3}{2}\right) \left(n - 2\right) \cdots \left(\frac{n}{2} + \frac{1}{2}\right)} \\ &= \lim_{n \rightarrow \infty} \frac{\cdots 3 \cdot 2 \cdot 1}{2^n \left(n - \frac{1}{2}\right) \left(n - \frac{3}{2}\right) \cdots \left(\frac{n}{2} + \frac{1}{2}\right)} \\ &= 0\end{aligned}$$